On Surface Integral Representations: Validity of Huygens' Principle and the Equivalence Principle in Inhomogeneous Bianisotropic Media

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Abstract—**Here we provide the mathematical foundation for general inhomogeneous (even discontinuous) media for the principle that Huygens devised with ingenious foresight over three hundred years ago (1690). We also validate the associated (and often used without a proof) equivalence principle as a natural extension of the isotropic formalism.**

I. Introduction

SURFACE integral representations (SIR) are very valuable for analytical and numerical investigations of scattering and radiation problems, mainly because they provide a foundation for approximations, and because the computer requirements are significantly reduced as compared with a volumetric approach.

Huygens' original idea that, in a homogeneous isotropic medium, each point on the wavefront could be regarded as a new source of waves, was extended by himself 300 years ago [1] to the case of the uniaxial calcite crystal in order to explain double refraction. This resulted in ingenious and beautiful geometric constructions, which clearly illustrate the validity of Huygens' principle for the uniaxial case, at least on an informal level, since polarization and amplitudes were not accounted for.

Application of Green's theorem provided a mathematical foundation for Huygens' principle in a homogeneous isotropic medium, but the uniaxial case was never validated. It is interesting to note that not even the inhomogeneous isotropic case has been treated in the literature in this context, and that until recently [2] a detailed mathematical formulation of the equivalence principle under the constraints of homogeneity and isotropy was unavailable (only a descriptive and pictorial presentation of the equivalence principle was available). However, the apparently more complex moving homogeneous isotropic media (a special type of bianisotropic) was successfully analyzed by Schwiger and Levine (see for instance [3]) who obtained a corresponding SIR. Also, the case of an inhomogeneous (but continuous) magnetoionic (plasma) medium was successfully treated by Williams [4] who obtained the SIR for the electric field. Two-dimensional problems involving homogeneous anisotropies have been simplified through the use of an SIR derived by this author for ten- [5] and (full tensor) eighteen- [6] parameter material; the former (simpler) case did not involve polarization coupling, while the latter involves depolarization and is more complex in nature. More recently, this author has derived SIR for the homogeneous biisotropic case [7].

This paper is apparently ambitious since we attempt to formally extend Huygens' principle to bianisotropic regions (i.e., anisotropic regions whose material media becomes polarized when placed in a magnetic field and magnetized when placed in an electric field) of arbitrary inhomogeneity, including discontinuities. However, as will be seen shortly, the analysis is rather simple and straightforward, leading to easy understanding of the principle.

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II. The Surface Integral Representations

Some confusion arises in the literature because some researchers employ different constitutive relations, which can be shown to be equivalent for time-harmonic fields. This is particularly true in the isotropic chiral reciprocal case (a very specialized subset of the general problem we are presently dealing with), as noted in [8]. Here we adopt constitutive relations corresponding to the model due to Tellegen [9], (also appearing in [10]), which is consistent with the model proposed by Post [11].

Let region V be a source-free inhomogeneous bianisotropic region where Maxwell's equations acquire the form

$$\nabla \times \overline{E} = -j\omega\left\{\overline{\overline{\mu}}(\overline{x}) \cdot \overline{H} + \overline{\overline{\zeta}}(\overline{x}) \cdot \overline{E}\right\} \tag{1a}$$

$$\nabla \times \overline{H} = j\omega\left\{\overline{\overline{\epsilon}}(\overline{x}) \cdot \overline{E} + \overline{\overline{\xi}}(\overline{x}) \cdot \overline{H}\right\}, \tag{1b}$$

where an $\exp(j\omega t)$ time convention is assumed and suppressed. Let V be bounded by surface S of inward normals $\hat{n}$ shown in Fig. 1. We now define the auxiliary fields $\overline{E}_a$, $\overline{H}_a$, which are point source responses in the complementary medium [10], for which $\overline{\overline{\mu}}^C = \overline{\overline{\mu}}^T$, $\overline{\overline{\epsilon}}^C = \overline{\overline{\epsilon}}^T$, $\overline{\overline{\zeta}}^C = -\overline{\overline{\xi}}^T$ and $\overline{\overline{\xi}}^C = -\overline{\overline{\zeta}}^T$:

$$\nabla \times \overline{E}_a = -j\omega\left\{\overline{\overline{\mu}}^T(\overline{x}) \cdot \overline{H}_a - \overline{\overline{\xi}}^T(\overline{x}) \cdot \overline{E}_a\right\} - C_m\hat{\ell}_m\delta(\overline{x} - \overline{x}') \tag{2a}$$

$$\nabla \times \overline{H}_a = j\omega\left\{\overline{\overline{\epsilon}}^T(\overline{x}) \cdot \overline{E}_a - \overline{\overline{\zeta}}^T(\overline{x}) \cdot \overline{H}_a\right\} + C_e\hat{\ell}_e\delta(\overline{x} - \overline{x}'') \tag{2b}$$

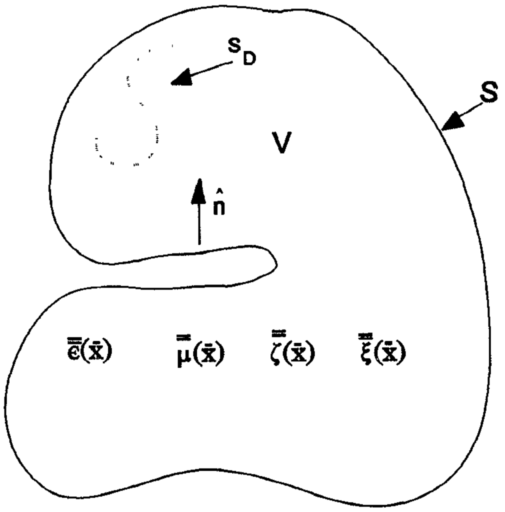

Fig. 1. Bianisotropic volume V is bounded by surface S (of inward normals $\hat{n}$) and is characterized by position-dependent material parameters, which are allowed to be discontinuous (on surface S_D).

where the superscript T denotes transpose, and C_m and C_e are the amplitudes of the magnetic and electric currents that are located in V at $\bar{x}'$ and $\bar{x}''$ respectively. $\hat{\ell}_m$ and $\hat{\ell}_e$ denote the corresponding orientations.

Dot multiplying (2b) by $\bar{E}$ and (1a) by $\bar{H}_a$, and after substracting the resulting equations and using the fact that $\bar{a} \cdot \bar{\bar{E}} \cdot \bar{b} = \bar{b} \cdot \bar{\bar{E}}^T \cdot \bar{a}$, we get:

$$\nabla \cdot \left(\bar{H}_a \times \bar{E}\right) = j\omega\left[\bar{E}_a \cdot \bar{\bar{\epsilon}}(\bar{x}) \cdot \bar{E} + \bar{H}_a \cdot \bar{\bar{\mu}}(\bar{x}) \cdot \bar{H}\right] + C_e \bar{E} \cdot \hat{\ell}_e \delta(\bar{x} - \bar{x}'') \tag{3a}$$

Similarly, from (2a) and (1b) we get:

$$\nabla \cdot \left(\bar{H} \times \bar{E}_a\right) = j\omega\left[\bar{E}_a \cdot \bar{\bar{\epsilon}}(\bar{x}) \cdot \bar{E} + \bar{H}_a \cdot \bar{\bar{\mu}}(\bar{x}) \cdot \bar{H}\right] + C_m \bar{H} \cdot \hat{\ell}_m \delta(\bar{x} - \bar{x}') \tag{3b}$$

Substructing the above two equations we obtain:

$$\nabla \cdot \left(\bar{H} \times \bar{E}_a - \bar{H}_a \times \bar{E}\right) = C_m \bar{H} \cdot \hat{\ell}_m \delta(\bar{x} - \bar{x}') - C_e \bar{E} \cdot \hat{\ell}_e \delta(\bar{x} - \bar{x}'') \tag{3c}$$

We can now apply Gauss's Theorem to (3c), over the volumes not containing the neighborhood of surfaces S_D, over which we find material discontinuities (abrupt change, interface), and where there is a chance that the argument of the divergence will be discontinuous. We obtain for $\bar{x}'$, $\bar{x}''$ in V but not on S_D:

$$\oint_S d\bar{S} \cdot \left(\bar{H} \times \bar{E}_a - \bar{H}_a \times \bar{E}\right) + \sum_\ell \int_{S_{D,\ell}} d\bar{S} \cdot \left[\bar{H} \times \bar{E}_a - \bar{H}_a \times \bar{E}\right]_-^+ = C_m \bar{H}(\bar{x}') \cdot \hat{\ell}_m - C_e \bar{E}(\bar{x}'') \cdot \hat{\ell}_e \,, \tag{4}$$

where the bracket $[\,]_-^+$ signifies jump across S_D. Had the points $\bar{x}'$ or $\bar{x}''$ not lain inside of V, their contribution to the right hand side of (4) would have been zero.

Because only the normal to S_D component of $(\bar{H} \times \bar{E}_a - \bar{H}_a \times \bar{E})$ plays a significant role, and because this only involves the tangential to S_D components of $\bar{H}$, $\bar{E}_a$, $\bar{H}_a$ and $\bar{E}$, which are known to be continuous across S_D, independently (provided of course no delta-function-like material jump occurs on S_D, such as can be caused by an infinitesimally thin resistive sheet), it follows that the integrals over S_D are zero, leading to

$$-C_m \bar{H}(\bar{x}') \cdot \hat{\ell}_m + C_e \bar{E}(\bar{x}'') \cdot \hat{\ell}_e = \oint_S dS \left\{(\hat{n} \times \bar{H}) \cdot \bar{E}_a - \bar{H}_a \cdot (\bar{E} \times \hat{n})\right\}. \tag{5}$$

By properly setting the C parameters to 1 and 0, and accounting for arbitrary source orientation $\hat{\ell}$ via the introduction of Dyadic Green's functions, we obtain the SIR.

A. SIR for the Electric Field

Here we set $C_m = 0$ and $C_e = 1$, and :

$$\bar{E}_a(\bar{x}) = \bar{\bar{G}}^c_{ee}(\bar{x}; \bar{x}'') \cdot \hat{\ell}_e$$
$$\bar{H}_a(\bar{x}) = \bar{\bar{G}}^c_{me}(\bar{x}; \bar{x}'') \cdot \hat{\ell}_e \,, \tag{6}$$

which define the $\bar{\bar{G}}^c$ dyadics in complementary space, whose first subindex refers to the field that is being calculated, whereas the second subindex refers to the kind of source (e/m = electric/magnetic).

Use of the above in (2), and after some manipulations results in the following differential equation for $\bar{\bar{G}}^c_{ee}(\bar{x}; \bar{x}'')$:

$$\nabla \times \left[\left(\bar{\bar{\mu}}^T\right)^{-1} \cdot \nabla \times \bar{\bar{G}}^c_{ee}\right] + j\omega\left\{\bar{\bar{\zeta}}^T \cdot \left(\bar{\bar{\mu}}^T\right)^{-1} \cdot \nabla \times \bar{\bar{G}}^c_{ee} - \nabla \times \left(\left(\bar{\bar{\mu}}^T\right)^{-1} \cdot \bar{\bar{\xi}}^T \cdot \bar{\bar{G}}^c_{ee}\right)\right\} - \omega^2\left[\bar{\bar{\epsilon}}^T - \bar{\bar{\zeta}}^T \cdot \left(\bar{\bar{\mu}}^T\right)^{-1} \cdot \bar{\bar{\xi}}^T\right] \cdot \bar{\bar{G}}^c_{ee} = -j\omega \bar{\bar{1}} \delta(\bar{x} - \bar{x}''), \tag{7}$$

where the position dependence of all the electric parameters is not shown explicitly for convenience. Further, from (6) and (2a) we find that $\bar{\bar{G}}^c_{me}(\bar{x}; \bar{x}'')$ is given by:

$$\bar{\bar{G}}^c_{me} = \left(\bar{\bar{\mu}}^T\right)^{-1} \cdot \left\{\frac{j}{\omega} \nabla \times \bar{\bar{G}}^c_{ee} + \bar{\bar{\xi}}^T \cdot \bar{\bar{G}}^c_{ee}\right\}. \tag{8}$$

Finally, use of (6) in (5) and eliminating the $\hat{\ell}_e$ factor results in:

$$\bar{E}(\bar{x}') = \oint_S dS \left\{(\hat{n} \times \bar{H}) \cdot \bar{\bar{G}}^c_{ee}(\bar{x}; \bar{x}') - (\bar{E} \times \hat{n}) \cdot \bar{\bar{G}}^c_{me}(\bar{x}; \bar{x}')\right\}, \tag{9}$$

which is the desired SIR. This, however, contains the foreign dyadics of the complementary space and therefore does not constitute a basis for definition of Huygens' principle nor can it be used to state an equivalence theorem. To accomplish this we need to identify source terms and express the above dyadics in terms of the real space ones.

Given the complexity of (7), the lack of reciprocity in this medium, and the fact that the G's are not functions of just $(\overline{x}-\overline{x}')$ (see, for instance, [5]), it is not deemed appropriate to use an algebraic method to try to establish some symmetry relations between the G's of the complementary medium and the corresponding ones of the real space. These relations are needed in order to be able to write the right-hand side of (9) on a more aesthetic form wherein the sources $\left(\hat{n}\times\overline{H}\text{ or }\overline{E}\times\hat{n}\right)$ and the response of the medium are clearly identified. A simple method of obtaining such relationships is presented in Section III.

B. SIR for the Magnetic Field

One obvious SIR for $\overline{H}$ is obtained by expressing $\overline{H}$ in terms of $\overline{E}$ and its curl from (1a), and using (5) for $\overline{E}$. Such an expression is, however, not too appealing because the sources of the fields are not evident as in (9). We now set $C_e=0$ and $C_m=1$, and

$$\overline{E}_a=\overline{\overline{G}}^c_{em}(\overline{x};\overline{x}')\cdot\hat{\ell}_m$$
$$\overline{H}_a=\overline{\overline{G}}^c_{mm}(\overline{x};\overline{x}')\cdot\hat{\ell}_m\,. \tag{10}$$

Following the steps that led to (9), we find that $\overline{\overline{G}}^c_{mm}(\overline{x};\overline{x}')$ satisfies:

$$\nabla\times\left[\left(\overline{\overline{\epsilon}}^T\right)^{-1}\cdot\nabla\times\overline{\overline{G}}^c_{mm}\right]+j\omega\left\{\nabla\times\left(\left(\overline{\overline{\epsilon}}^T\right)^{-1}\cdot\overline{\overline{\zeta}}^T\cdot\overline{\overline{G}}^c_{mm}\right)-\overline{\overline{\xi}}^T\cdot\left(\overline{\overline{\epsilon}}^T\right)^{-1}\cdot\nabla\times\overline{\overline{G}}^c_{mm}\right\}-\omega^2\left[\overline{\overline{\mu}}^T-\overline{\overline{\xi}}^T\cdot\left(\overline{\overline{\epsilon}}^T\right)^{-1}\cdot\overline{\overline{\zeta}}^T\right]\cdot\overline{\overline{G}}^c_{mm}=-j\omega\overline{\overline{1}}\delta(\overline{x}-\overline{x}') \tag{11}$$

while $\overline{\overline{G}}^c_{em}(\overline{x};\overline{x}')$ is related to $\overline{\overline{G}}_{mm}(\overline{x};\overline{x}')$ via:

$$\overline{\overline{G}}^c_{em}=\left(\overline{\overline{\epsilon}}^T\right)^{-1}\cdot\left[\frac{1}{j\omega}\nabla\times\overline{\overline{G}}^c_{mm}+\overline{\overline{\zeta}}^T\cdot\overline{\overline{G}}^c_{mm}\right], \tag{12}$$

and the resulting SIR is given by

$$\overline{H}(\overline{x}')=-\oint_S dS\left\{\left(\hat{n}\times\overline{H}\right)\cdot\overline{\overline{G}}^c_{em}(\overline{x};\overline{x}')-\left(\overline{E}\times\hat{n}\right)\cdot\overline{\overline{G}}^c_{mm}(\overline{x};\overline{x}')\right\}. \tag{13}$$

Once again we observe that no Huygens' principle or physical interpretation can be given to this representation because no relationship between the complementary and real space Green's dyadic is available. These relationships are established in the next section, and, as will be seen, they appear to be deceptively simple.

III. Huygens' Principle

A simple yet powerful method of relating $\overline{\overline{G}}$ and $\overline{\overline{G}}^c$ is provided by the modified reciprocity theorem for inhomogeneous bianistropic regions [10]. According to this reaction theorem, the reaction of sources $\overline{J}_A$[electric: $\overline{J}_A=J_A\hat{\ell}^e_A\delta(\overline{x}-\overline{x}^e_A)$] and M_A (magnetic: $\overline{M}_A=M_A\hat{\ell}^m_A\delta(\overline{x}-\overline{x}^m_A)$) caused by sources $\overline{J}_B$ (electric: $\overline{J}_B=J_B\hat{\ell}^e_B\delta(\overline{x}-\overline{x}^e_B)$) and $\overline{M}_B$[magnetic: $\overline{M}_B=M_B\hat{\ell}^m_B\delta(\overline{x}-\overline{x}^m_B)$] is equal to the reaction of sources $\overline{J}_B$, $\overline{M}_B$ caused by $\overline{J}_A$, $\overline{M}_A$ in the complementary medium. This can be written as:

$$\int_\infty dV\left(\overline{J}_A\cdot\overline{E}_B-\overline{M}_A\cdot\overline{H}_B\right)=\int_\infty dV\cdot\left(\overline{J}_B\cdot\overline{E}^C_A-\overline{M}_B\cdot\overline{H}^C_A\right) \tag{14}$$

where the superscript C stands for complementary space, and the subscripts A and B in the fields stand for the sources. Using the expressions for the sources, (14) reduces to

$$J_A\hat{\ell}^e_A\cdot\overline{E}_B(\overline{x}^e_A)-M_A\hat{\ell}^m_A\cdot\overline{H}_B(\overline{x}^m_A)=J_B\hat{\ell}^e_B\cdot\overline{E}^C_A(\overline{x}^e_B)-M_B\hat{\ell}^m_B\cdot\overline{H}^C_A(\overline{x}^m_B)\,. \tag{15}$$

Furthermore, and in agreement with (6) and (10), we have that the fields are given by:

$$\overline{E}_B(\overline{x}^e_A)=\overline{\overline{G}}_{ee}(\overline{x}^e_A;\overline{x}^e_B)\cdot\hat{\ell}^e_BJ_B+\overline{\overline{G}}_{em}(\overline{x}^e_A;\overline{x}^m_B)\cdot\hat{\ell}^m_BM_B \tag{16a}$$

$$\overline{H}_B(\overline{x}^m_A)=\overline{\overline{G}}_{me}(\overline{x}^m_A;\overline{x}^e_B)\cdot\hat{\ell}^e_BJ_B+\overline{\overline{G}}_{mm}(\overline{x}^m_A;\overline{x}^m_B)\cdot\hat{\ell}^m_BM_B \tag{16b}$$

and

$$\overline{E}^C_A(\overline{x}^e_B)=\overline{\overline{G}}^c_{ee}(\overline{x}^e_B;\overline{x}^e_A)\cdot\hat{\ell}^e_AJ_A+\overline{\overline{G}}^c_{em}(\overline{x}^e_B;\overline{x}^m_A)\cdot\hat{\ell}^m_AM_A \tag{17a}$$

$$\overline{H}^c_A(\overline{x}^m_B)=\overline{\overline{G}}^c_{me}(\overline{x}^m_B;\overline{x}^e_A)\cdot\hat{\ell}^e_AJ_A+\overline{\overline{G}}^c_{mm}(\overline{x}^m_B;\overline{x}^m_A)\cdot\hat{\ell}^m_AM_A\,. \tag{17b}$$

Use of (16)–(17) in (15) leads to:

$$J_AJ_B\left\{\hat{\ell}^e_A\cdot\overline{\overline{G}}_{ee}(\overline{x}^e_A;\overline{x}^e_B)\cdot\hat{\ell}^e_B-\hat{\ell}^e_B\cdot\overline{\overline{G}}_{ee}(\overline{x}^e_B;\overline{x}^e_A)\cdot\hat{\ell}^e_A\right\}+J_AM_B\left\{\hat{\ell}^e_A\cdot\overline{\overline{G}}_{em}(\overline{x}^e_A;\overline{x}^m_B)\cdot\hat{\ell}^m_B+\hat{\ell}^m_B\cdot\overline{\overline{G}}^c_{me}(\overline{x}^m_B;\overline{x}^e_A)\cdot\hat{\ell}^e_A\right\}+M_AM_B\left\{-\hat{\ell}^m_A\cdot\overline{\overline{G}}_{mm}(\overline{x}^m_A;\overline{x}^m_B)\cdot\hat{\ell}^m_B+\hat{\ell}^m_B\cdot\overline{\overline{G}}^c_{mm}(\overline{x}^m_B;\overline{x}^m_A)\cdot\hat{\ell}^m_A\right\}-M_AJ_B\left\{\hat{\ell}^m_A\cdot\overline{\overline{G}}_{me}(\overline{x}^m_A;\overline{x}^e_B)\cdot\hat{\ell}^e_B+\hat{\ell}^e_B\cdot\overline{\overline{G}}^c_{em}(\overline{x}^e_B;\overline{x}^m_A)\cdot\hat{\ell}^m_A\right\}=0\,. \tag{18}$$

From this and the fact that the source amplitudes as well as their locations are arbitrary, we find that the G^c and G are related via

$$\overline{\overline{G}}^c_{ee}(\overline{x};\overline{x}')=\overline{\overline{G}}^T_{ee}(\overline{x}';\overline{x}) \tag{19a}$$

$$\overline{\overline{G}}^c_{me}(\overline{x};\overline{x}')=-\overline{\overline{G}}^T_{em}(\overline{x}';\overline{x}) \tag{19b}$$

$$\overline{\overline{G}}^c_{mm}(\overline{x};\overline{x}')=\overline{\overline{G}}^T_{mm}(\overline{x}';\overline{x}) \tag{19c}$$

$$\overline{\overline{G}}^c_{em}(\overline{x};\overline{x}')=-\overline{\overline{G}}^T_{me}(\overline{x}';\overline{x})\,. \tag{19d}$$

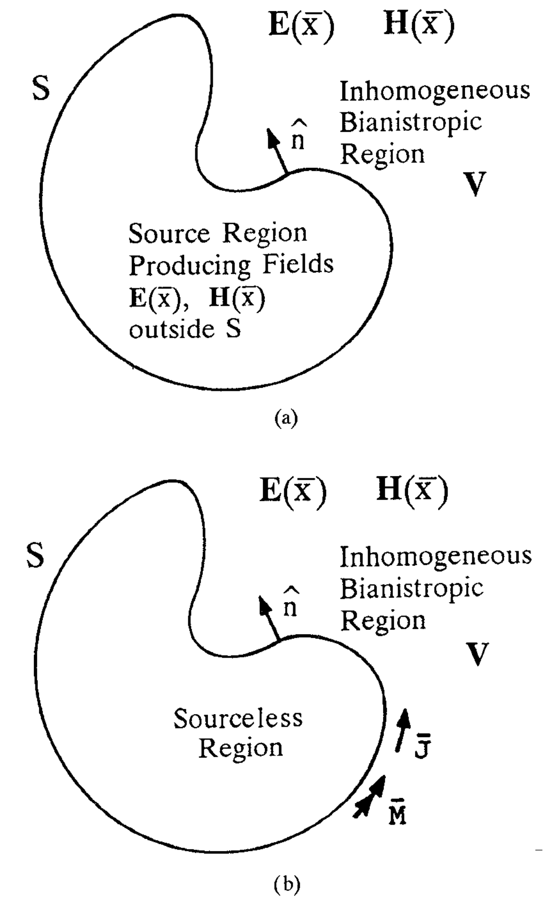

Fig. 2. Illustration of Huygens' principle for inhomogeneous bianisotropic regions. (a) Original configuration. (b) Equivalent configuration. As in the homogeneous isotropics case: $\overline{J} = \hat{n} \times \overline{H}$, $\overline{M} = \overline{E} \times \hat{n}$.

A simple set of relationships whose similarity to the ones characterizing the complementary medium parameters in terms of the real space parameters is to be noted.

Use of (19) in (9) and (13), and after interchanging $\overline{x}'$ and $\overline{x}$, and defining:

$$\overline{J}_S = \hat{n} \times \overline{H}; \qquad \overline{M}_S = \overline{E} \times \hat{n}, \tag{20}$$

we obtain:

$$\oint_S dS' \left\{ \overline{\overline{G}}_{ee}(\overline{x};\overline{x}') \cdot \overline{J}_S(\overline{x}') + \overline{\overline{G}}_{em}(\overline{x};\overline{x}') \cdot \overline{M}_S(\overline{x}') \right\} = \overline{E}(\overline{x}) \tag{21}$$

$$\oint_S dS' \left\{ \overline{\overline{G}}_{me}(\overline{x};\overline{x}') \cdot \overline{J}_S(\overline{x}') + \overline{\overline{G}}_{mm}(\overline{x};\overline{x}') \cdot \overline{M}_S(\overline{x}') \right\} = \overline{H}(\overline{x}) \tag{22}$$

for $\overline{x} \in V$. Equations (21) and (22) state that the field solution in the inhomogeneous bianisotropic domain V is completely determined by the tangential fields of $\overline{E}$ and $\overline{H}$ specified over the surface S enclosing V. Therefore, equations (21) and (22) clearly validate the extension of the usual homogeneous isotropic Huygens' principle to inhomogeneous bianisotropic domains. This is illustrated in Fig. 2.

IV. The Equivalence Principle

If we had accounted for sources $\overline{J}$ and $\overline{M}$ in region V in the previous discussion [i.e., include source terms on the right hand side of (1)], and allow the point $\overline{x}$ to lie outside V as well [as discussed in the sentence following (4)], we could have found that the integrals on the left-hand side of (21) and (22) represent only the scattered field. In the presence of sources in V, (21) and (22) become

$$\overline{E}^{\,\mathrm{inc}}(\overline{x}) + \oint_S dS' \left\{ \overline{\overline{G}}_{ee}(\overline{x};\overline{x}') \cdot \overline{J}_S(\overline{x}') + \overline{\overline{G}}_{em}(\overline{x};\overline{x}') \cdot \overline{M}_S(\overline{x}') \right\} = \begin{cases} \overline{E}(\overline{x}); & \overline{x} \in V \\ 0; & \overline{x} \notin V \end{cases} \tag{23}$$

$$\overline{H}^{\,\mathrm{inc}}(\overline{x}) + \oint_S dS' \left\{ \overline{\overline{G}}_{me}(\overline{x};\overline{x}') \cdot \overline{J}_S(\overline{x}') + \overline{\overline{G}}_{mm}(\overline{x};\overline{x}') \cdot \overline{M}_S(\overline{x}') \right\} = \begin{cases} \overline{H}(\overline{x}); & \overline{x} \in V \\ 0; & \overline{x} \notin V, \end{cases} \tag{24}$$

where $\overline{E}^{\,\mathrm{inc}}(\overline{x})$, $\overline{H}^{\,\mathrm{inc}}(\overline{x})$ are the fields produced by the sources $\overline{J}$ and $\overline{M}$ in free (inhomogeneous) infinite space.

From (23) and (24) we arrive at the almost paradoxic (but sound) conclusion that the representation is valid in V, regardless of how we "continue" or extend our inhomogeneous material (defined only in V) to regions outside V. Similarly, for $\overline{x} \notin V$, (23) and (24) establish relationships between the tangential fields on the boundary; the relationships being obviously unique, independently of the form of the $\overline{\overline{G}}$'s, which can vary as the material can be arbitrarily extended to regions outside V.

The above, somewhat strange concept, is not so strange, since it even holds under more familiar isotropic conditions. As a simple illustration of this we consider the spherical shell region of Fig. 3(a). Here V is bounded by the spherical surfaces S_a and S_b, and contains the sources $\overline{J}$, $\overline{M}$. Let for simplicity V be characterized by homogeneous biisotropic ϵ, μ, γ and β parameters, so that a field expansion in terms of solutions to the Helmhotz equations (for both modes) is feasible [7], resulting in a clearly valid relationship between the tangential fields on S_a and S_b, independently of what materials we have outside V. Now consider the situation of Fig. 3(b), where an anisotropic sphere is embedded in volume V_{core} (biisotropic, same as V), which is in turn bounded by S_a. Consider for simplicity the case where the anisotropy $\overline{\overline{\epsilon}}$, $\overline{\overline{\mu}}$ is such that allows decomposition of the fields in terms of (transversal θ, ϕ) Tesseral harmonics, such as occurs in [12]. It is then feasible to formally and straightforwardly solve the problem of Fig. 3(b), in the inhomogeneous region: $V + V_{\mathrm{core}}$, obtaining a representation at any point in terms of the tangential fields on S_b alone. Thus, the fields tangential to any surface such as S_a are expressible in terms of the tangentials on S_b, such a relationship being independent of what lies inside V_{core}. This simple example shows that a Green's dyadic accounting for scattering by bodies or inhomogeneities outside V may be employed as well in a field representation in V.

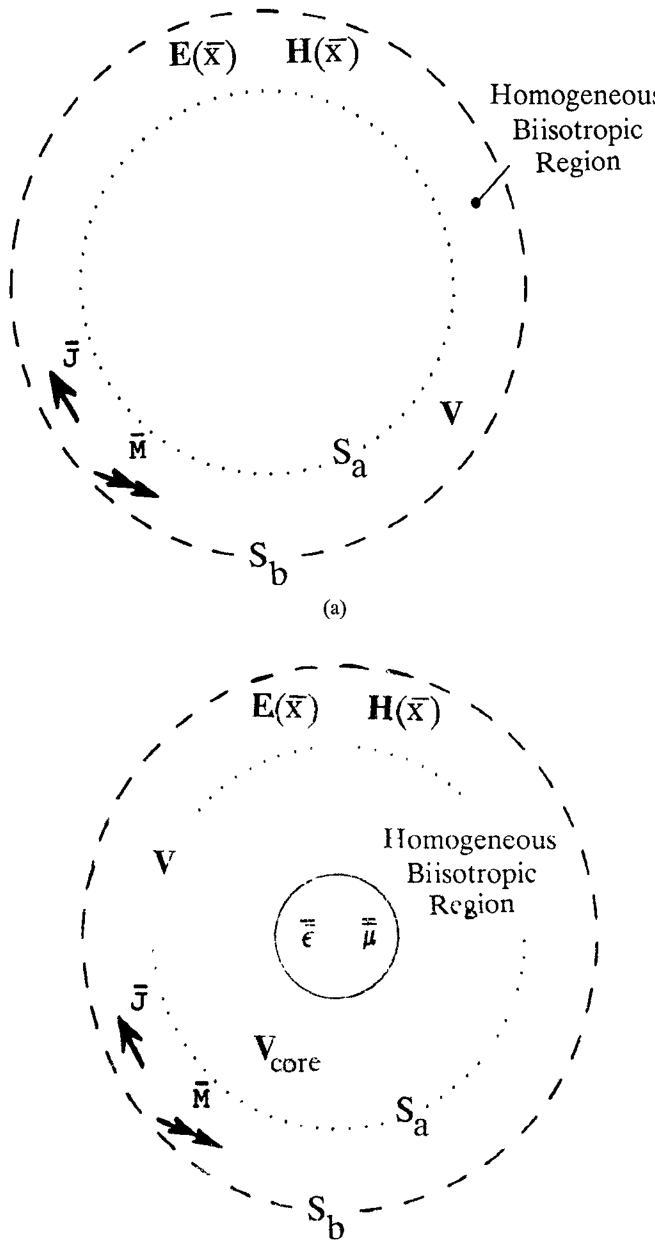

Fig. 3. A simple illustration of the fact that the field representation in valid in V, regardless of how the inhomogeneities are extended outside of V. (a) Homogeneous biisotropic spherical shell region. Via a standard model solution a relationship between the tangential fields on S_a and S_b is established. (b) Here V is an inhomogeneous region consisting of the anisotropic core and the surrounding biisotropic space. A model solution is also feasible and incorporates the effect of the core; the relationship between the tangential or S_a and S_b will, however, remain unchanged. This demonstrates that a problem in a given region may be posed in terms of point source responses, which correspond to arbitrary inhomogeneities outside the regions of interest.

Equations (23) and (24) are in standard form and since they involve only the Green's dyadics in real space, they clearly validate the applicability of the homogeneous isotropic equivalence principle for inhomogeneous bianisotropic regions. The equivalent currents do not need to be modified in any form with respect to those in the isotropic case and, likewise, they radiate in "free" space. The use of (23)–(24) is illustrated in Fig. 4.

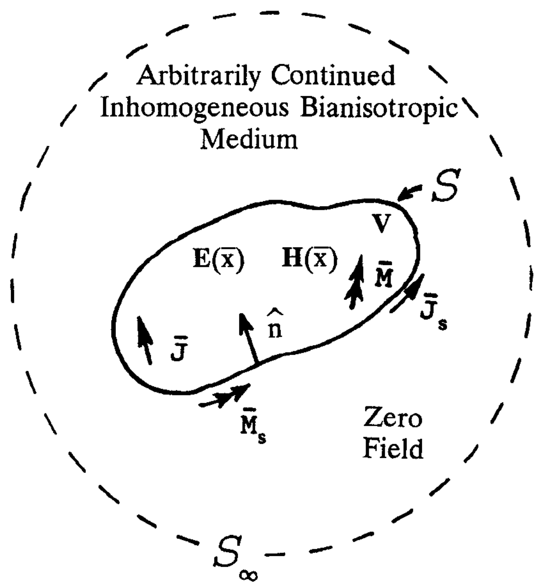

Fig. 4. Pictorial illustration of (23)–(24), as applied to the inhomogeneous bianisotropic region V characterized by $\epsilon(\overline{x})$, $\overline{\overline{\mu}}(\overline{x})$, $\overline{\overline{\xi}}(\overline{x})$, and $\overline{\overline{\zeta}}(\overline{x})$. The material outside V is arbitrary (inhomogeneous), and has no effect in the computation of fields $\overline{E}(\overline{x})$, $H(\overline{x})$, in V, in terms of the equivalent surface currents $\overline{J}_S$, $\overline{M}_S$. The elements $\overline{J}$ and $\overline{M}$ in V represent sources.

In view of the fact that (23)–(24) are identical in form to those encountered in the homogeneous isotropic analyses [2], it seems trivial and unnecessary to go into a detailed description of the equivalence principle, by considering outer and inner regions to show that in V the effective currents can be written as $\overline{J}_{\text{eq}} = \hat{n} \times (\overline{H}_{\text{in}} - \overline{H}_{\text{out}})$, $\overline{M}_{\text{eq}} = (\overline{E}_{\text{in}} - \overline{E}_{\text{out}}) \times \hat{n}$. Suffice it to show the results in graphical form. This shown in Fig. 5, which applies (23)–(24) to a region external to that considered in Fig. 4 (surface at infinity does not contribute due to radiation behavior of dyadic Green's functions provided inhomogeneities do not allow unattenuated wave guidance to infinity). Combining Figs. 4 and 5 we obtain the familiar field representation in V. This is shown in Fig. 6, where net equivalent currents are defined. The tangentials $\overline{H}_{\text{out}}$ and $\overline{E}_{\text{out}}$ have no real effect on the field representation (they are however related through the field equations in the external to V region) and can be chosen so as to cancel the effect of either $\overline{H}_{\text{in}}$ or $\overline{E}_{\text{in}}$ or partly $\overline{H}_{\text{in}}$ and partly $\overline{E}_{\text{in}}$. For convenience this is illustrated is Fig. 7 and we do not elaborate on this since the isotropic concept remains unchanged.

V. Conclusion

Huygens' principle and the equivalence principle have been shown to be applicable to bianisotropic inhomogeneous regions. Just as in the isotropic case, the equivalent currents involve only the tangential fields $(\hat{n} \times \overline{H}, \overline{E} \times \hat{n})$, and radiate in free (infinite) space. The definite noninvolvement of the normal field components (as occurs in the presence of diffusion) is another attractive feature of this exact formalism and may lead to many potential applications, for the number of unknowns in any interaction problem is substantially reduced (to the bounding surface). Even though our result appears deceptively simple, the basic physical processes such

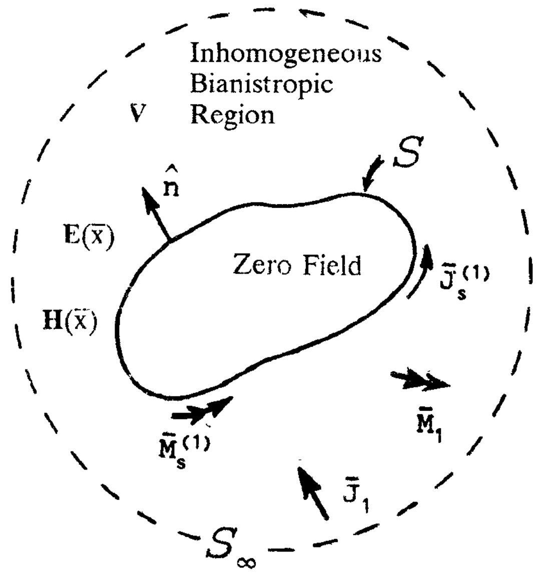

Fig. 5. Representation (23)–(24) applied to volume V bounded by the closed surface S and the spherical surface of infinite radius S_∞. Provided the inhomogeneities in V are such that do not allow localized, guided (unantenuated) transport of energy to infinity (such as could happen with a straight optical fiber extending to infinity), the surface at infinity gives no contribution to the surface integral. Here $\overline{J}_S^{(1)}$ and $\overline{M}_S^{(1)}$ are equivalent sources on S, and $\overline{J}_1$, $\overline{M}_1$ are actual sources. The region internal to S contains an arbitrary medium (inhomogeneous, bianisotropic).

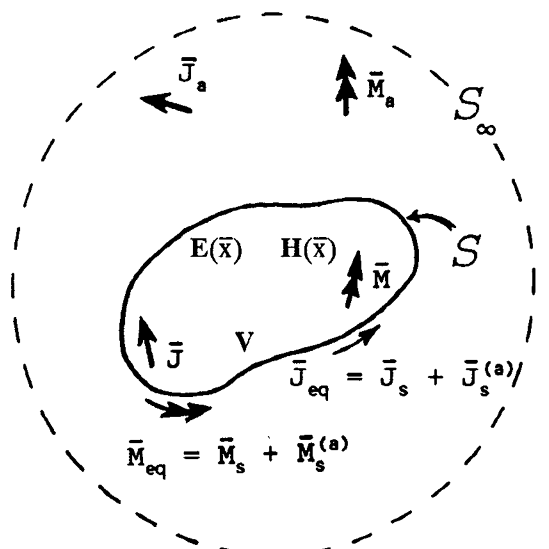

Fig. 6. Combination of the models of Figs. 4 and 5, in a composite field representation in volume V. The elements corresponding to the volume external to V are denoted by an "a", for auxiliary fields. The entire space is bianisotropically inhomogeneous, reducing to the prescribed inhomogeneities within V. The net equivalent sources are $\overline{J}_{\text{eq}}$ and $\overline{M}_{\text{eq}}$.

as birefringence, lack of reciprocity, optical rotatory power, and extraordinary/inhomogeneous character of elemental wave behavior, and so on, are all properly accounted for. This study should lead to better understanding of physical processes in bianisotropic media.

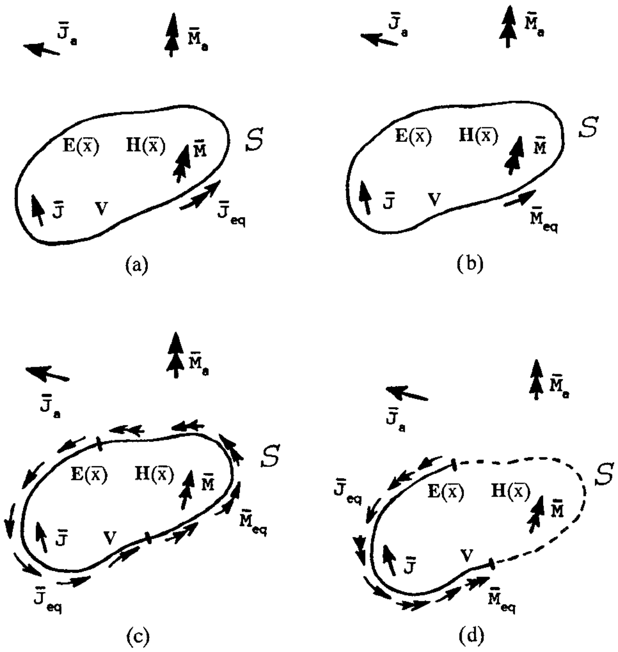

Fig. 7. The nonuniqueness of the equivalent sources (depicted in Fig. 6) can be exploited to generate an infinite number of equivalent models. Here we sketch a few: (a) using only electric currents; (b) using only magnetic currents; (c) partial electric, partial magnetic over complementary regions of S; (d) electric and magnetic over partial region of S only (half).

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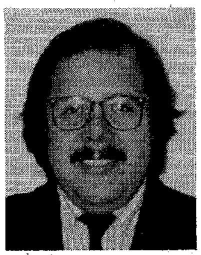
REFERENCES

[1] A. Sommerfield, Optics. NY: Academic Press, 1954, Section 27.
[2] K.-M. Chen, "A mathematical formulation of the equivalence principle," *IEEE Trans. Microwave Theory Tech.*, vol. 37, no. 10, pp. 1576–1581, Oct. 1989.
[3] C.T. Tai, Dyadic Green's Functions in Electromagnetic Theory, Scranton; Intext, 1971.
[4] R.H. Williams, "A field equivalence theorem for linear magnetoionic media," *IEEE Trans. Antennas Propagat.*, vol. AP-12, no. 4, pp. 511–512, July 1964.
[5] J.C. Monzon, "On a surface integral representation for homogeneous anisotropic regions; two-dimensional case," *IEEE Trans. Antennas Propagat.*, vol. 36, no. 10, pp. 1401–1406, Oct. 1988.
[6] ——, unpublished work, research performed at Damaskos, Inc.
[7] ——, "Radiation and scattering in general homogeneous biisotropic regions," *IEEE Trans. Antennas Propagat.*, vol. 38, no. 2, Feb. 1990, pp. 227–235.
[8] A. Lakhtakia, V. Varadan, and V.K. Varadan, "Field equations, Huygen's principle, integral equations, and theorems for radiation and scattering of electromagnetic waves in isotropic chiral media," *J. Opt. Soc. Amer.*, vol. A5, p. 175, 1988.
[9] B.D.H. Tellegen, "The gyrator: A new electric network element," Phillips Res. Rep., vol. 3, p. 81, 1948.
[10] J.A. Kong, Electromagnetic Wave Theory. NY: Wiley, 1985.
[11] E.J. Post, Formal Structure of Electromagnetics. Amsterdam: North-Holland, 1962.
[12] J.C. Monzon, "Three-dimensional field expansion in the most general rotationally symmetric anisotropic material: Application to scattering by a sphere," *IEEE Trans. Antennas Propagat.*, vol. 37, no. 6, pp. 728–735, June 1989.



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